VIBRATION ANALYSIS OF FUNCTIONALLY GRADED PLATES WITH MULTIPLE CIRCULAR CUTOUTS

Avadesh K Sharma[1], Premanand S. Chauhan [2], Manoj Narwaria[3], Rajkumar Pankaj[4]
[1]Asst. Prof., Department of Mechanical Engineering., MITS, Gwalior (M.P)
[2]Prof., Department of Mechanical Engineering, IPS-CTM, Gwalior (M.P)
[3]Associate Prof., Department of Mechanical Engineering, IPS-CTM, Gwalior (M.P)
[4]Research Scholar, Department of Mechanical Engineering, MGCGV, Chitrakoot, Satna (M.P)
Email: sharma.avadesh@gmail.com[1], prempunit@gmail.com[2],
manoj.narwariya@gmail.com[3],raj_mits2003@yahoo.co.in[4]

Abstract

This paper presents free vibration analysis of functionally graded plates with multiple circular cutouts. Finite element method is used to investigate the free vibration of functionally graded plates. The material properties of the plates are assumed to vary according to a power law distribution in terms of the volume fraction of the constituents. In this paper, the effects of volume fraction index, thickness ratio and different boundary conditions on the natural frequencies of plates is studied.

1. Introduction

In functionally graded materials, the volume fraction of two or more constitute materials are varied continuously as a function of position along certain dimension of the structure. The continuous change in the microstructure of the functionally graded materials advances them from the composite materials. The FGM is used in casing of sensors of the space shutle so that it can carry thermal as well as mechanical load. In FGM's these problems are avoided or gradually reduced by varying the volume fraction of constituents of FGM. The concept of FGM was first considered in Japan in 1984 during a space plane project, where a combination of materials used would serve the purpose of a thermal barrier capable of withstanding a surface temperature of 2000 K and a temperature gradient of 1000 K across a 10 mm section [1]. Science the concept of FGM was first proposed, FGM's are extensively studied by researchers, who have mainly focused on their static, dynamic and thermal behavior. The problems of free vibrations, wave propagation and static deformations in FGM beams were solved using an especially developed finite element method accounting for power law and other alternative variations of elastic and thermal properties in the thickness direction [2-3]. The modal employed a first order shear deformation theory of beam. Three methods were used for the static and dynamic analyses of square thick FGM plates with simply supported edges [4]. The methods employed in the paper included a higher order shear deformation theory and two novel solutions for FGM structures. According to this paper, the application of the normal deformation theory may be justified if the in-plane size to thickness is equal to or smaller than 5.

Researchers have also turned their attention to the vibration and dynamic response of FGM's structures [5-7]. Chen et al [8] presented exact solutions for free vibration analysis of rectangular plates using Bessel functions with three edges conditions. Liew et al [9] studied the free vibration analysis of functionally graded plates using the element-free Kp-Ritz method. They studied the free vibration analysis of four types of functionally graded rectangular and skew plates. Hiroyuki Matsunaga [10] presented in his paper, the analysis of natural frequencies and buckling of FGM's plates by taking into account the effects of transverse shear and normal deformations and rotary inertia. Atashipour et al [11] presented a new exact closed- form procedure to solve free vibration analysis of FGM's rectangular thick plates based on the Reddy's third-order shear deformation plate theory.

For plates with cutouts, Chai [12] presented finite element and some experimental results on the free vibration of symmetric composite plates with central hole. Sakiyama and Huang [13] proposed an approximate method for analyzing the free vibration of square plate with different cutouts. Liu, et al [14] studied static and free vibration analyses of composite plates with different cutouts via a linearly conforming radial point interpolation method. Maziar and Iman [15] studied the effect of relative distance of cutouts and size of cutouts on natural frequencies of FG plates with cutouts. Sharma and Mittal [16-19] studied the free vibration analysis of laminated composite plates using finite element method.

From the review of the above literature it is observed that very little research and analysis work has been done yet on the natural frequencies of the FG plates with cutouts. The study presents here, the effect of volume fraction index, thickness ratio and different external boundary conditions on the natural frequencies of FG (Al/Al_2O_3) plates i.e. rectangular, trapezoidal and circular plates with circular cutouts.

2. Functionally Graded Material Properties

A functionally graded material plate as shown in Fig. 1 is considered to be a plate of uniform thickness that is made of ceramic and metal. The material property is to be graded through the thickness according to a Power-Law distribution that is

$$P(z) = P_m + (P_c - P_m)V_f, \qquad (1a)$$

$$V_f = (\frac{z}{h})^n \qquad (n \geq 0), \qquad (1b)$$

Where P represents the effective material property, $P_c \; and \; P_m$ denotes the ceramic and metal properties respectively, V_f is the volume fraction of the ceramic, h is the thickness of the plate, $0 \leq z \leq h$ and n is the volume fraction index.

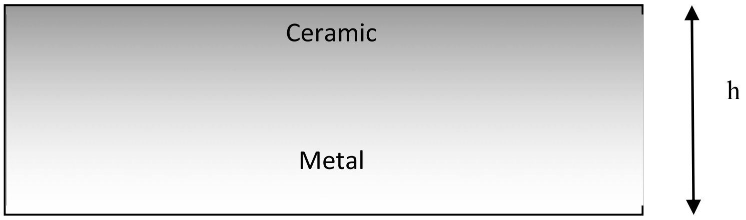

Fig. 1: Functionally Graded Plate

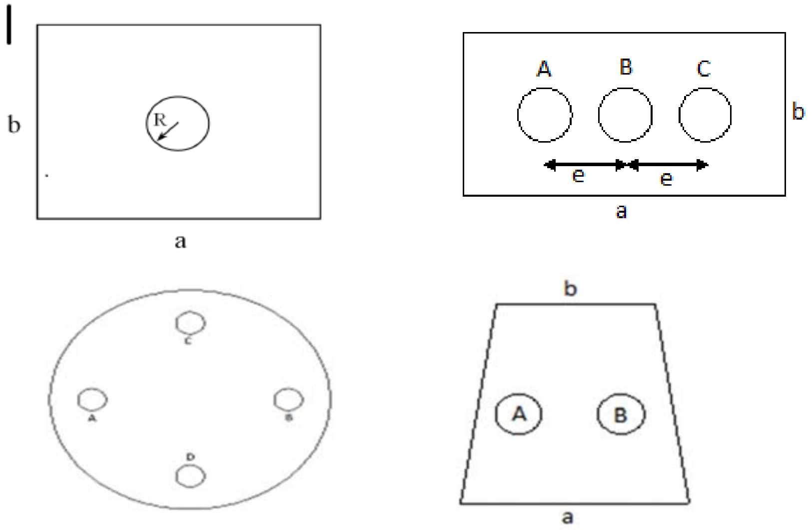

Fig. 2: Different Geometries of a functionally graded plates with circular cutouts

3. Functionally Graded Plate Elements

The finite element software (ANSYS) is used with the aim of analyzing. In addition SOLID 185 is used for modeling general 3-D solid structures. It allows for prism and tetrahedral degenerations when used in irregular regions. The element is defined by eight nodes having three degree of freedom at each node. More than 2000 nodes might be used in calculation work

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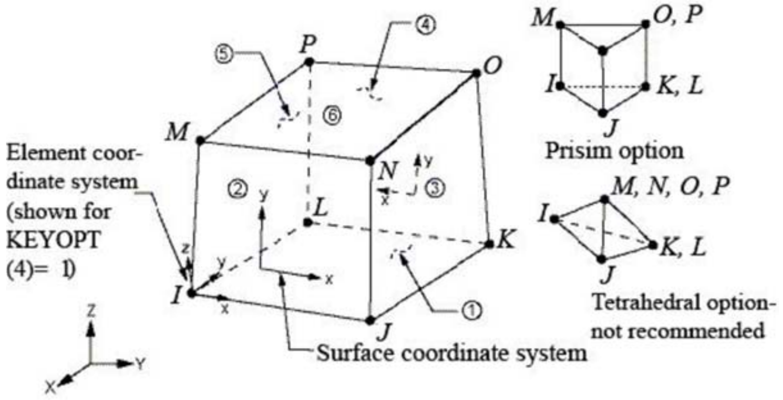

Fig. 3: SOLID 185

4. Numerical Results and Discussion

The material properties used in the convergence study are as follows:

$E_t = 70\ GPa,\quad v_t = 0.3,\quad \rho_t = 3800\ Kg/m^3,$
$\alpha_t = 7.4 \times 10^{-6}/°\mathrm{C},\quad K_t = 65\ W/mk$
$E_b = 70\ GPa,\ v_b = 0.3,\quad \rho_b = 2700\ Kg/m^3,$
$\alpha_b = 7.4 \times 10^{-6}/°\mathrm{C},\quad K_b = 65\ W/mk$

In order to show the accuracy of methodology used for free vibration analysis of FG plates with cutouts, the fundamental natural frequencies of different plates (such as rectangular, trapezoidal and circular plates with circular cutouts) are compared with the solutions presented by Maziar and Iman [15].

4.1 Isotropic plates

A square plate with a circular cutout at the center is shown in Fig. 2, the length of this plate is a=10, the ratio of the radius to length is r/a = 0.1 and the thickness ratio is h/a = 0:01. The material properties are the same as above. Table 1 provides a comparison between present results and solution given by Huang and Sakiyama [13] and Liu et al [14] for a simply supported plate.

Table 1: Nondimensional frequencies of isotropic square plate with square cutout at the center (simply support for external boundaries, $\varpi = [\rho\omega^2 a^4/(D(1 - \nu^2))]$, h/a = 0.01).

Mode	Present	Liu et al [14]	Huang et al [13]
1	4.540	6.149	6.240
2	7.223	8.577	8.457
3	7.231	8.634	8.462
4	9.124	10.42	10.23
5	10.23	11.41	11.72
6	10.56	11.84	12.30
7	11.79	12.83	13.04
8	11.80	12.84	13.04
9	13.54	-	-
10	13.56	-	-

Table 2 shows the comparison of natural frequencies of FG rectangular plate of side ratio a/b=2 having three holes of radius ratio r/b=0.15 and centre to centre distance ratio e/b= 0.7.

Table 2: Comparison of natural frequencies of FG rectangular plate with three circular cutouts (fully clamped for external boundaries)

	Mode									
	1	2	3	4	5	6	7	8	9	10
e/b										
0.7	496.99	607.47	931.21	1135.2	1221.3	1268.9	1436.0	1719.8	1727.1	1850.5
Iman[15]	500.45	614.97	931.33	1161.6	1249.9	1286.0	1458.8	1777.2	1800.5	2103.7

Table 3 shows the comparison of natural frequencies of FG circular plate of same radius and thickness with four holes of radius r= 0.1 at location A(x/R=-0.7, y/R=0), B(x/R=0.7, y/R=0), C(x/R=0, y/R=0.7) and D(x/R=0, y/R=-0.7) respectively.

Table 3: Comparison of natural frequencies of FG circular disc with four circular cutouts (fully clamped for external boundaries)

	Mode									
	1	2	3	4	5	6	7	8	9	10
e/b										
0.7	209.46	435.34	437.12	699.64	718.95	847.69	1032.8	1038.8	1267.6	1278.5
[15]	201.00	418.02	418.72	671.82	688.25	780.02	987.20	988.24	1180.6	1184.8

Table 4 shows the variation of natural frequencies of Al/Al_2O_3 FG rectangular plates with two circular holes (clamped-simply supported for external boundaries) with different thickness ratio of the plate as h/b=0.04, 0.06 and 0.08 respectively. The results for first ten modes are computed. For the FG plates with fully clamped external boundary condition, the frequencies in all ten modes decreases as the volume fraction index increases. This is expected, because a large volume fraction index means that a plate has a smaller ceramic component and thus its stiffness is reduced.

Table 4: Variation of natural frequencies with the volume fraction index *n* for Al/Al_2O_3 FG rectangular plates with two circular holes (clamped-simply supported for external boundaries)

		Mode									
		1	2	3	4	5	6	7	8	9	10
h/b	n										
0.04	0	947.24	1224.8	1626.3	2258.2	2268.8	2491.6	2904.2	3121.6	3471.2	3915.7
	0.5	820.68	1016.9	1351.7	1871.3	1908.9	2022.8	2459.4	2595.7	2935.5	3250.9
	1	656.15	838.38	1102.7	1529.6	1568.3	1693.9	2001.8	2126.1	2449.1	2676.4
	2	657.42	814.09	1065.0	1446.6	1544.4	1588.6	2025.9	2141.1	2375.7	2440.4
0.06	0	1323.4	1705.4	2227.5	3068.9	3099.0	3310.3	3935.5	4103.8	4223.3	4675.4
	0.5	1077.2	1348.3	1763.3	2423.2	2515.5	2643.2	3209.5	3332.1	3726.4	3774.7
	1	886.91	1082.2	1434.1	1970.5	2028.0	2155.2	2608.4	2737.3	3121.3	3379.1
	2	720.36	909.95	1191.0	1648.5	1671.2	1773.8	2135.5	2297.3	2560.3	2804.4
0.08	0	1675.1	2129.4	2790.8	3709.6	3848.1	4064.1	4124.2	4819.7	5168.5	5341.2
	0.5	1292.8	1620.9	2118.5	2910.1	2932.2	3129.5	3734.0	3776.9	4032.5	4493.6
	1	1005.4	1273.3	1682.3	2315.7	2331.2	2514.7	2971.7	3213.9	3403.9	3551.8
	2	884.57	1094.8	1448.1	1978.4	1991.1	2133.8	2513.1	2773.8	2878.7	3008.4

Table 5 shows the variation of natural frequencies of Al/Al_2O_3 FG trapezoidal plates with two circular holes (clamped-simply supported for external boundaries) with different thickness ratio of the plate is h/b=0.04, 0.06 and 0.08 respectively. The results for first ten modes are computed. For the FG plates with fully clamped external boundary condition, the frequencies in all ten modes decreases as the volume fraction index increases. This is expected, because a large volume fraction index means that a plate has a smaller ceramic component and thus its stiffness is reduced

.**Table 5:** Variation of natural frequencies with the volume fraction index *n* for Al/Al_2O_3 FG trapezoidal plates with two circular holes (clamped-simply supported for external boundaries)

		Mode									
		1	2	3	4	5	6	7	8	9	10
h/a	n										
0.04	0	2137.5	3888.6	4489.3	5591.4	6346.1	6612.5	7048.6	7454.2	7561.0	8406.7
	0.5	1581.1	2956.3	3381.0	4249.1	4831.1	5756.8	5965.9	6365.0	6537.0	6779.1
	1	1213.9	2294.9	2616.8	3342.3	3835.7	4563.0	5144.5	5400.3	5480.4	5800.6
	2	1035.7	1931.1	2179.7	2813.5	3147.1	3764.5	4353.4	4401.8	4586.3	4891.1
0.06	0	2716.8	4861.4	5622.0	6709.5	6848.4	7130.4	7627.8	7726.7	8731.4	9117.9
	0.5	2097.2	3785.4	4427.5	5537.2	6036.1	6199.2	6421.1	6892.1	7346.8	7859.9
	1	1640.5	2987.4	3454.8	4348.0	4879.5	5459.9	5767.2	5839.2	6252.0	6643.5
	2	1350.7	2459.8	2799.9	3477.2	3941.7	4585.9	4691.1	4896.5	5249.9	5323.3
0.08	0	3241.9	5743.5	6625.7	6819.6	7190.7	7725.4	7955.9	8843.2	8933.1	9474.2
	0.5	2533.2	4452.6	5150.2	6080.4	6346.6	6457.9	6932.7	7141.5	7951.8	8373.3
	1	2070.4	3733.9	4240.1	5266.4	5510.2	5845.2	5972.7	6285.2	6982.7	7161.5
	2	1616.5	2898.5	3308.3	4034.6	4578.9	4606.5	4893.3	5265.2	5331.4	5943.9

The variation of first ten natural frequencies with the thickness ratio is depicted in Fig. 4.

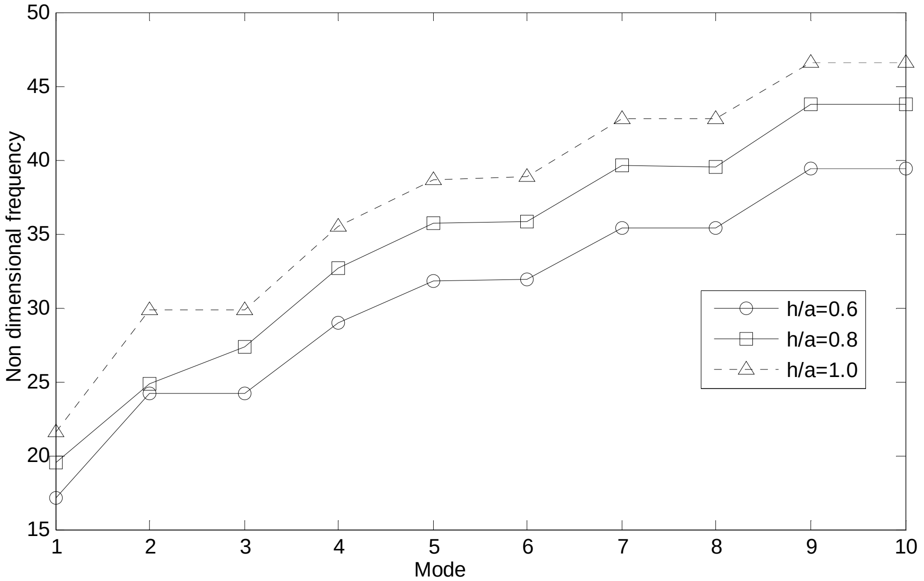

Fig.4: Effect of variation of thickness ratio on the first ten natural frequencies of square plate

5. Conclusions

In this paper, free vibration behavior of functionally graded plates with circular cutouts has been carried out using APDL code in ANSYS and results are validated with the available published results. The elastic properties of FG plates are assumed to vary through the thickness according to a power law. It is found that a volume fraction exponent that ranges between 0 and 1 has a significant influence on the natural frequency of FG plates with cutouts. For rectangular, trapezoidal and circular FG plates with circular cutouts, the natural frequency decreases as the volume fraction index increases.

References

[1] Functionally Graded Materials (FGM) and Their Production Methods”, Azom.com.

[2] A. Chakraborty, S. Gopalakrishnan and J.N. Reddy, “A New Beam Finite Element for the Analysis of Functionally Graded Materials”, Int. J. of Mechanical Science 45 (2003), pp. 519-539.

[3] A. Chakraborty and S. Gopalakrishnan, “A Spectrally Formulated Finite Element for Wave Propagation Analysis in Functionally Graded Beam”, Int. J. of Solids and Structures, 40 (2003), pp. 2421-2448.

[4] L. F. Qian, R.C. Batra and L. M. Chen, “Static and Dynamic Deformation of Thick Functionally Graded Elastic Plates by Using Higher Order Shear and Normal Deformable Plate Theory and Meshless Local Petrov-Galerkin Method”, Composites, Part B 35 (2005), pp. 685-697.

[5] H.L. Dai, X. Wang, “Thermo-Electro-Elastic Transient Responses in Piezoelectric Hollow Structures”, Int. J. of Solids and Structures 42 (2005), pp. 1151-1171.

[6] X. L. Huang, S. H. Shen, “Nonlinear Vibration and Dynamic Response of Functionally Graded Plates in Thermal Environments”, Int. J. of Solids and Structures 41 (2004), pp. 2403-2427.

[7] J. Yang, S. H. Sheen, “Free Vibration and Parametric Response of Shear Deformable Functionally Graded Cylindrical Panels”, Journal of Sound and Vibration 261 (2003), pp. 871-893.

[8] Jiu Hui Wu, A.Q. Liu and H. L. Chen, “Exact Solutions for Free Vibration Analysis of Rectangular Plate using Bessel Functions”, Journal of Applied Mechanics 74 (2007), pp. 1247-1251.

[9] X. Zhao, Y. Y. Lee and K. M. Liew, “Free Vibration Analysis of Functionally Graded Plates using the Element-Free Kp-Ritz Method”, Int. J. of Sound and Vibration 319 (2009), pp. 918-939.

[10] Hiroyuki Matsunaga, “Free Vibration and Stability of Functionally Graded Plates According to a 2-D Higher-Order Deformation Theory”, Int. J. of Composite Structures 82 (2008), pp. 499-512.

[11] Ah. Hosseini- Hashemi, M. Fadaee and S. R. Atashipour, “Study on the Free Vibration of Thick Functionally Graded Rectangular Plates According to a New Exact Closed-Form Procedure”, Int. J. of Composite Structures 93 (2011), pp. 722-735.

[12] B. G. Chai, “Free Vibration of Laminated Plates with a Central Circular Hole”, Journal of Composite Structures 35 (1996), pp. 357-368.

[13] T. Sakiyama, M. Huang, “Free Vibration Analysis of Rectangular Plates with Variously Shape-Hole”, Journal of Sound and Vibration 226 (1996), pp. 769-786.

[14] G. R. Liu, X. Zhao, K.Y. Dai, Z.H. Zhong, G. Y. Li and X. Han, “Static and Free Vibration Analysis of Laminated Composite Plates using the Conforming Radial Point Interpolation Method”, Int. J. of Composites Science and Technology 68 (2008), pp. 354-366.

[15] J. Maziar and R. Iman, “Free Vibration Analysis of Functionally Graded Plates with Multiple Circular and Non-circular Cutouts”, Chinese Journal of Mechanical Engineering 24 (2011).

[16] Sharma A.K., Mittal N.D., Review on stress and vibration analysis of composite plates, Journal of Applied Sciences, 10(23), 2010, 3156-3166.

[17] Sharma A.K., Mittal N.D., Free vibration analysis of laminated composite plates with elastically restrained edges using FEM, Central European Journal of Engineering, 3(2) (2013) 306-315.

[18] Sharma, A.K. & Mittal, N.D., Sharma A., Free vibration analysis of moderately thick antisymmetric cross-ply laminated rectangular plates with elastic edge constraints. International Journal of Mechanical Sciences, 53, 2011, 688–695.

[19] Sharma, A. K. & Mittal, N.D., Free vibration analysis of moderately thick Anti-symmetric angle ply laminated rectangular plates with elastic edge constraints. Mechanics of Advanced Materials and Structures, 21, 2014, 341–348

[20] Rutishauser H.,"Simultaneous Iteration Method for Symmetric Matrices", Int. J. of Numerische Mathematik 16(1970), pp. 205-223.

[21] Rutishauser H., "Computational Aspects of F. L. Bauer's Simultaneous Iteration Method", Int. J. of Numerische Mathematik 13 (1969), pp. 4-13.

[22] Bathe K. J., "The subspace Iteration Method-Revisited", Int. J. of Computers and Structures (2012).